ASSESSMENT AND CONTROL OF HUMAN INDUCED VIBRATIONS IN THE NEW COIMBRA FOOTBRIDGE

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Summary

The paper describes a dynamic study of a new slender footbridge currently under construction at the city of Coimbra, in Portugal. The bridge displays innovative characteristics and a complex dynamic behaviour. Numerical studies indicate the vulnerability to lateral and vertical vibrations induced by pedestrians, and the need to install control devices. A set of TMDs is proposed, and the corresponding efficiency analysed, considering the various uncertainties present in the analysis.

Keywords: Footbridge; Vibration assessment; TMD; Control of vibrations; Crowd Loads

1. Introduction

The development of the Mondego Green Park along both banks of the river at the city of Coimbra, Portugal, set the opportunity to construct a pedestrian and cycling bridge that is intended to be a landmark for the city and to contribute to the quality of a new leisure area.

An innovative bridge was designed [1,2] by Adao da Fonseca, leading the engineering team from AFAssociados (www.afaconsultores.pt), designated in this paper as the Structural Engineer, and by Cecil Balmond, leading the architectural team from Ove Arup (www.arup.com). The bridge is very slender, with a total length of 275m and a width of 4m, formed by a central parabolic arch and two half parabolic arches in steel supporting in total continuity a composite steel-concrete deck (Figure 1).

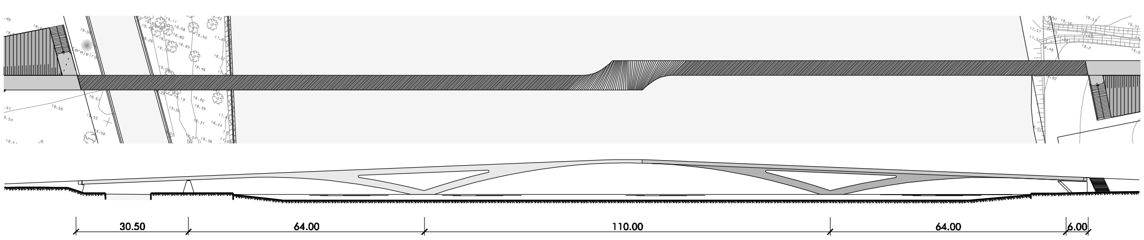


Fig. 1 - Plan view and elevation of the footbridge

Preliminary dynamic studies by AFAssociados indicated that the bridge is prone to vibrations induced by pedestrians and requires control measures. A detailed numerical study of the bridge was then performed in order to better characterise the dynamic behaviour under the action of pedestrians and to identify the required control measures. Such study was developed by the Laboratory of Vibrations and Monitoring (VIBEST) from the Faculty of Engineering of the University of Porto (http://www.fe.up.pt/vibest), based on the current knowledge concerning the characterisation of human loads and the definition of vibration limits. It comprehended the development of a finite element model and the performance of a sensitivity analysis to better understand the dynamic behaviour of the bridge, followed by a set of numerical simulations of pedestrian loads, considering different scenarios and different levels of simplification: resonant response induced by a single pedestrian; resonant response induced by a group of pedestrians in synchronised or non-synchronised motion; and resonant response associated with a continuous flow of pedestrians with a defined level of synchronisation.

Those responses indicate that lateral synchronisation associated with the first lateral vibration is critical and show that high levels of vibration for several vertical vibration modes may develop. Therefore, the addition of tuned mass dampers (TMDs) for several modes was proposed, but a final decision on the effective need of all the suggested TMDs depends on the damping values to be measured on the constructed bridge. A simplified analysis was performed to study the efficiency of the association of those TMDs.

The present paper discusses the various uncertainties associated with: i) bridge mechanical characteristics, like the interaction of the light superstructure with the flexible foundations; ii) simplifications adopted in numerical modelling, considering for example the complex torsion behaviour; iii) impossibility to specify exact damping values; iv) definition of load models; and v) definition of limit response.

2. Characterisation of the bridge

A central parabolic arch and two lateral parabolic half arches in steel support with total continuity a composite steel-concrete deck. The central arch spans 110m and rises 9m. The anti-symmetry of both arch and deck cross-sections along the longitudinal axis of the bridge (as shown in Figure 1 and Figure 2) is a unique feature of this bridge. Complex torsion behaviour under vertical loads is generated but lateral stiffness is increased from that of the traditional structure with symmetrical cross-sections along the entire length of the bridge.

The arch rectangular box cross-section is 1.35m x 1.80m and the deck has an L-shaped box cross-section with its top flange formed by a composite steel-concrete slab 0.11m thick (Figure 2). In the central part of the bridge, each L-shaped box cross-section and corresponding arch "meet" and go forward to form a rectangular box cross-section 8m x 0.90m. In the lateral spans, arch and deck generate a rectangular box cross-section 4m x 0.90m.

A key factor for the bridge global stiffness is the structural behaviour of its foundations, which are formed by vertical piles deep 35m. Due to poor characteristics of the soil layers, piles are quite flexible under horizontal loads and the structural behaviour of the bridge is mixed arch and girder.

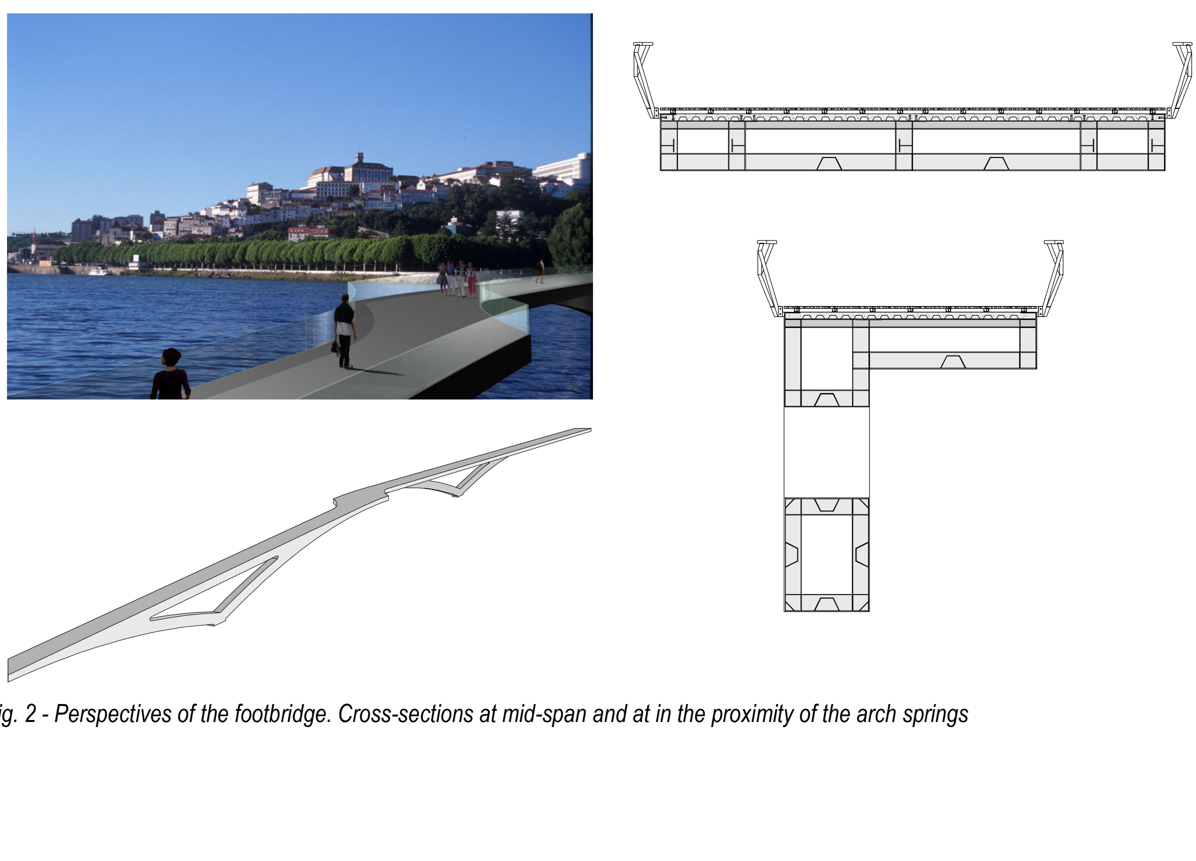


Fig. 2 - Perspectives of the footbridge. Cross-sections at mid-span and at in the proximity of the arch springs

3. Numerical model

The dynamic study was performed by a beam finite element (FE) model developed in the SOLVIA software. The FE model incorporates a series of simplifications whose effects were previously calibrated by the Structural Engineer with two other FE models (Figure 3): one three-dimensional shell model and one beam model, both with piles discretised into beam elements acted upon by nonlinear horizontal springs of variable stiffness, for distinct soil layers.

3.1 Assumptions

The SOLVIA FE model discretises deck and arches into beam elements whose axes contain the centre of gravity of the corresponding cross-sections. Therefore, the resulting mesh presented in Figure 3 displays some irregularity. Cross-section mechanical characteristics were provided by the Structural Engineer and refer to the orthogonal axes parallel to the bridge global axes in the centre of mass.

Structural materials are steel S355 and concrete C35/45 (EC4) for the deck slab. The instantaneous value of the concrete Young Modulus has been considered in the definition of the modular ratio to evaluate equivalent steel sections.

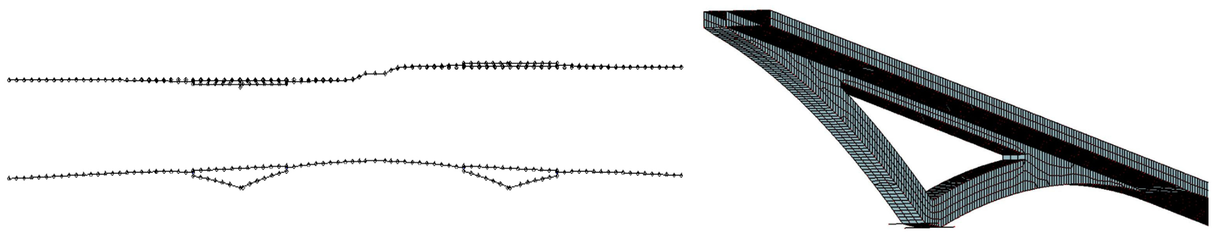


Fig. 3 - Plan and side view of the beam finite element mesh. Detail of shell finite element model

The arch foundation is assumed rigid for rotations and elastic for horizontal translations. Geotechnical information and modelling of soil-pile interaction by the Structural Engineer for frequent loads led to a stiffness constant of $K_x=K_y=100000$kN/m.

Steel dead-weight is applied at the box centre of mass, while deck loads are applied at the deck mid-line. A set of nodes located along this line were generated, connected to the bridge nodes with identical abscissas by rigid links. Deck mass is concentrated on those nodes.

3.2 Parametric studies

A sensitivity analysis was made on that model in order to evaluate the degree of geometric nonlinearity, the effect of the flexibility of the arch foundations, and the effect of the added mass of pedestrians into the bridge natural frequencies.

Table 1 contains global static response values and first natural frequencies determined in the following analyses: (i) LCP - Linear elastic analysis of bridge model with clamped arch foundations, submitted to permanent loads; (ii) GCP - Geometric nonlinear analysis of bridge model with clamped arch foundations, submitted to permanent loads; (iii) GFP - Geometric nonlinear analysis of bridge model with flexible foundations for horizontal displacements ($K_x=K_y=100000$kN/m), submitted to permanent loads; (iv) GFF - Geometric nonlinear analysis of bridge model with flexible foundations for horizontal displacements ($K_x=K_y=100000$kN/m), submitted to an added load of 1.2kN/m^2, with relation to previous calculations.

Table 1 shows that second order effects due to bridge deformation are negligible. On the contrary, flexibility of foundations affects greatly the behaviour of the bridge, almost doubling vertical displacements due to permanent loads and decreasing significantly the vertical stiffness. The reduction of frequency affects essentially vertical vibration modes that are approximately symmetric with respect to the vertical plane located at the bridge mid-span. This is the case of the 3rd bending mode, with natural frequency decreasing 31%.

Pedestrian mass was introduced by a uniform load of 1.2kN/m^2 that adds 16% to the bridge mass and reduces around 10% to all natural frequencies. It should be noted that those percentages are the result of added fixed masses to the bridge, but it is known that the effect of a moving mass is less significant.

Table 1 - Global response of the footbridge for different model analyses

Analysis / Type of response		LCP	GCP	GFP	GFF
Longitudinal displacement at arch spring (mm)		-	-	17.9	21.5
Lateral displacement at arch spring (mm)		-	-	3.0	3.4
Mid-span vertical displacement (mm)		125	129	240	291
Vertical stiffness (kN/m)*		8166	7480	4049	4049
Lateral stiffness (kN/m)*		4580	4544	4519	4519
Maximum tension force in the deck (kN)		3681	3702	4743	4743
Maximum compressive force in arch (kN)		6909	6963	6851	8111
Maximum bending moment in arch (kN.m)		11797	11971	13758	16555
Frequency (Hz)	1st mode	0.783	0.775	0.765	0.704
	2nd mode	1.42	1.37	0.942	0.854
	3rd mode	1.21	1.19	1.182	1.083
	4th mode	1.64	1.62	1.519	1.403

* Calculated as the necessary concentrated load applied at mid-span to produce a unit displacement at that point in the relevant direction

3.3 Modal parameters

Modal parameters in this dynamic study refer to analyses designated above as GFP and GFF, with pedestrian effects calculated for a single pedestrian or group of pedestrians (model GFP) or for a continuous flow of pedestrians (model GFF). Table 2 displays the characteristics of modal configurations for the first 18 vibration modes. Figure 4 represents plan and side views of the most relevant modal configurations.

Natural frequencies in the 3D-model developed by the Structural Engineer with pedestrian loading as in the GFP model are also presented in Table 2. Conclusion is that the beam model is in good agreement with the more refined model.

Due to the three-dimensional behaviour of the bridge, most modes involve simultaneous lateral and vertical displacements. The dominant displacement is shown in bold in Table 2. The first four modes are also classified as anti-symmetric (ASM) or symmetric (SYM). However, it should be noted that this classification is only approximate, since the bridge is not effectively symmetric with respect to mid-span.

Table 2 shows that many natural frequencies lay within the range of frequencies considered as critical in terms of pedestrian excitation. Two modes (numbers 1 and 4) are critical for the lateral direction and eight modes (numbers 5 and 7 to 14) are critical for the vertical direction.

Damping ratio is a critical parameter in the investigation of the response to pedestrian loads. A variation within the range 0.5% to 2% should be expected, according to literature references to steel and composite footbridges. In the calculations, a constant value of 1% is used for all modes.

4. Evaluation of pedestrian effects

The existence of various vibration modes in the frequency range typically excited by pedestrians, for lateral and vertical directions, motivated a detailed numerical analysis of those effects, in which various conditions of excitation were simulated, considering different levels of simplification. The calculated response was compared with limit values defined in international codes and literature in order to evaluate the necessity to introduce control measures.

4.1 Definition of load models

No methodology for the analysis of pedestrian load effects is suggested in international codes, but a common procedure is to consider ultimate limit state combinations of actions for static effects of pedestrian loads in the calculations and to analyse the corresponding dynamic effects in terms of comfort through the specification of vibration limits.

In the evaluation of those effects, different degrees of complexity of methodologies of analysis can be considered. That depends on the importance of the bridge and on its proneness to pedestrian induced vibrations. Accordingly, three levels of loading can be specified: single pedestrian, pedestrian group and crowd loading. For each level, various procedures can be followed, as discussed in the following subsections.

Table 2 - Natural frequencies and characteristics of modal shapes

Mode number	Shell FEM GFP	Frequency GFP (Hz)	Frequency GFF (Hz)	Characteristic of vibration mode (Shell)
1	0.64	0.765	0.704	**Lateral SYM**
2	0.91	0.942	0.854	**Vertical SYM**
3	1.12	1.182	1.083	**Vertical ASM**
4	1.3	1.519	1.403	Vertical SYM + **Lateral ASM**
5	1.64	1.692	1.548	**Vertical**
6	1.67	1.836	1.703	**Lateral**
7	2.01	2.191	2.001	**Vertical**
8	2.08	2.435	2.261	**Vertical** + Lateral
9	2.23	2.526	2.336	Vertical+**Lateral**
10	2.27	2.602	2.375	**Vertical**
11	2.45	2.683	2.498	**Vertical**
12	2.77	2.736	2.522	**Vertical**
13	3.22	2.832	2.705	**Vertical**+Lateral (**Vertical**)
14	3.26	3.321	3.060	**Vertical (Torsion)**
15	3.45	3.931	3.625	**Vertical**+Lateral (Vertical+ **Lateral**)
16	3.88	4.153	3.847	Vertical+**Lateral (Vertical)**
17	4.23	4.633	4.260	**Vertical**+Lateral
18	4.89	5.157	4.768	Vertical+**Lateral** (**Torsion**+ Vertical)

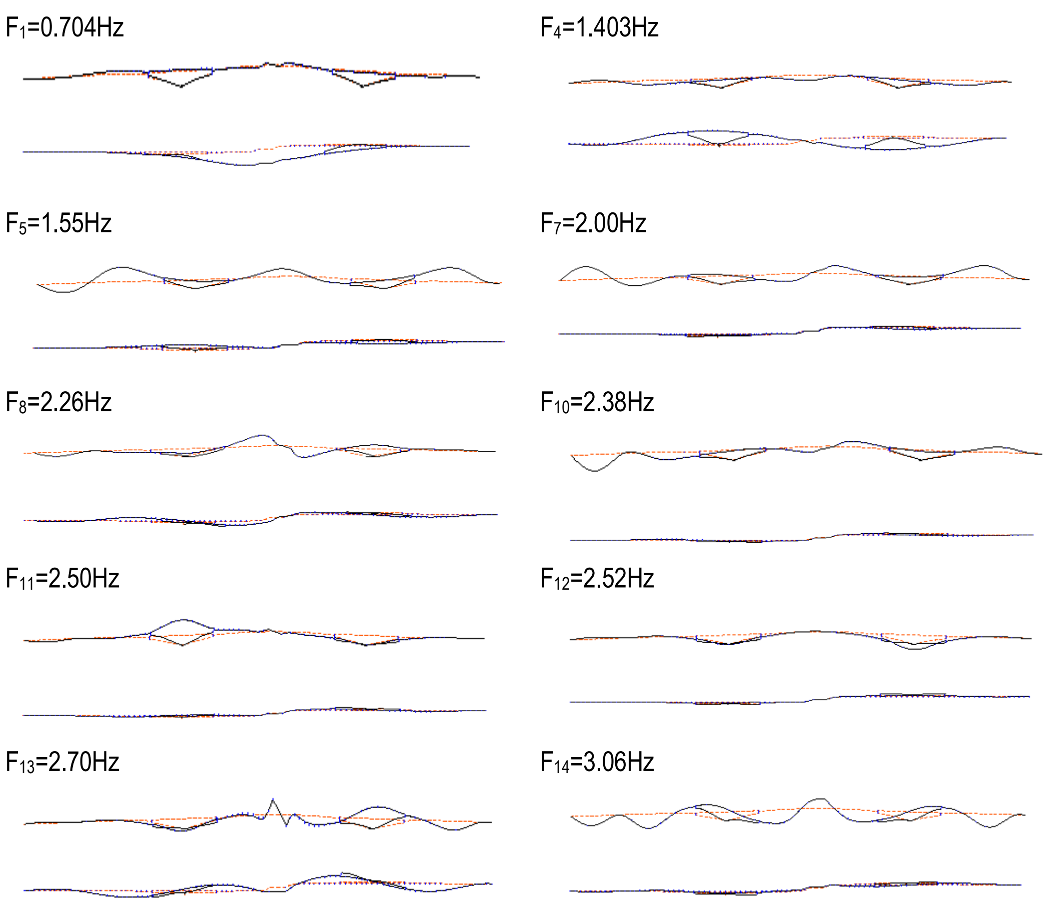


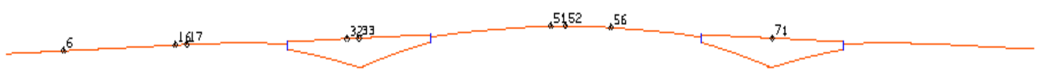
Fig. 4 - Configurations of critical vibration modes for pedestrian excitation - side (top) and plan view (bottom)

4.2 Simulation of pedestrian effects

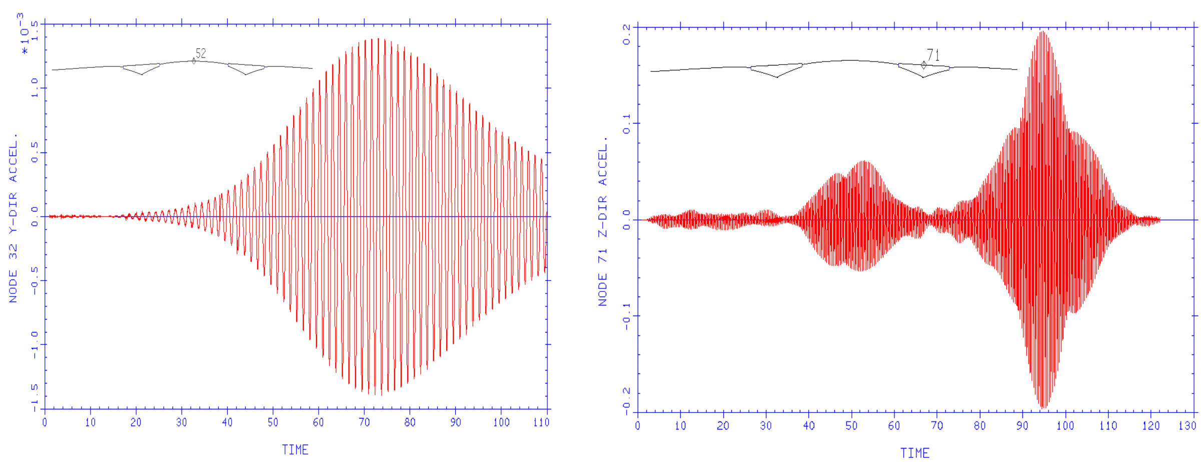
4.2.1 Single pedestrian

The analysis of single pedestrian response can be made in a simplified manner, with the pedestrian located on a fixed position along the bridge and with a sinusoidal excitation of natural frequency equal to the bridge fundamental frequency. In this case, assuming the dynamic behaviour of the bridge to be linear, either spectral analysis or direct time integration analysis provide the required response. A more realistic approach defines a concentrated load according to a specified load model [3,4], and that load is moved along the bridge considering the velocity of propagation and step length of the pedestrian. The response is then calculated by direct time integration.

Table 3 - Bridge maximum response associated with single pedestrian excitation



Mode number	Natural frequency (Hz)	Antinode	Spectral response, fixed location		Time integration, moving load	
			Disp.(mm)	Accel. (cm/s²)	Disp.(mm)	Accel. (cm/s²)
1	0.704	52Y	0.75	1.44	0.62	1.22
4	1.403	32Y			0.26	1.88
5	1.548	17Z	0.76	7.19		
7	2.001	6Z	0.53	8.41		
8	2.261	56Z	0.13	2.57		
10	2.375	6Z	0.79	17.7	0.78	15.8
11	2.498	33Z	0.61	15.1		
12	2.522	71Z	0.64	16.1	0.82	19.6
13	2.705	51Z	0.094	2.73	0.11	3.3
14	3.060	16Z	0.14	5.00		



(a) (b)

Fig. 5 - Response history associated with single pedestrian propagation: (a) lateral acceleration at mid-span for f_p=0.704Hz; (b) vertical acceleration at mid-point of tie for f_p=2.522Hz (m/s²)

All three approaches were followed in this study, in order to value the degree of approximation of the simplest methodology. The investigated vibration modes are highlighted in Table 2. Considering a pedestrian weight of 800N and a frequency of pedestrian excitation f_p equal to the bridge natural frequency f_0 (or twice the bridge natural frequency, for the lateral direction Y, if $f_0 \leq 1.25$Hz), a concentrated sinusoidal force defined by $280 \times \sin(2\pi f_p t)$ was applied in the vertical direction Z at the antinode of each vertical vibration mode and a horizontal force of $80 \times \sin(\pi f_p t)$ (or $40 \times \sin(\pi f_p t)$, if $f_0 > 2.5$Hz) was applied at the antinode of each lateral vibration mode. The response was calculated for the relevant direction at the antinode of each mode using both direct time integration and modal superposition analysis, and spectral analysis. Since the formulations were based on the same principles, results coincide. The simulation of pedestrian propagation was then performed by generating a concentrated load formed by vertical and lateral components defined as stated above, the vertical component including the second and third harmonic terms, according to Bachmann load model [3]. The two component load propagates with a velocity defined by $0.9xf_p$. Table 3 summarises the maximum values of displacement and acceleration at the bridge antinodes considering both the spectral approach for fixed location pedestrian and the two components moving load.

Figure 5 displays the time histories of the mid-span lateral response and of the vertical response at the mid-point of the deck span over the arch spring when a single pedestrian crosses the bridge with natural frequency equal to 2x0.704Hz or 2.522Hz, respectively.

It can be seen that the fixed load approximation provides a conservative estimate of the lateral response. In the vertical direction, slightly higher displacements and accelerations may result as consequence to the response contribution to lateral and vertical harmonic terms associated with the chosen propagation frequency.

4.2.2 Small group of pedestrians

Group loads depend on the bridge size and on the frequency of the excitation. Small groups of 2 to 6 persons can synchronise for relatively low walking frequencies. For higher frequencies, associated with running or jogging, synchronisation in the group is difficult and, for higher number of elements, say 10 to 15, should no longer be expected, except if intentional excitation ("vandal loads") is considered. Taking into account those possibilities, group response calculations can be achieved either by simplified formulae extrapolating single pedestrian response or by simulation of a number of moving concentrated loads generated according to a specific law.

In this study, the simplified approach of synchronised motion of a group of 13 persons was followed. The calculated response is summarised in Table 4 for the critical vibration modes, expressed in terms of maximum acceleration and displacement, a_{gmax} and d_{gmax}, respectively. This hypothesis is rather conservative because walking of a group at natural frequencies higher than 2Hz corresponds to fast walk. However, if the possibility of intentional excitation by jump is considered, the amplitude of excitation increases by a factor of about 4 and therefore the calculated response corresponds approximately to that produced by a group of 3 vandals in full synchronised motion.

4.2.3 Continuous flow of pedestrians

The characterisation of action induced by a continuous flow of pedestrians has a certain degree of incertitude, motivated by the possibility of feedback phenomena that arises as consequence of high levels of induced oscillation, and results in a not well-defined degree of synchronisation among pedestrians. To the author's knowledge, feedback phenomena have been clearly observed for lateral vibrations but not for vertical vibrations. In reality, human perception to lateral vibrations is more intense than to vertical vibrations, not only because of physiological aspects related with equilibrium but also because of the distinct rate of occurrence. Therefore, although synchronisation can be expected for the vertical direction, the triggering condition for its occurrence implies higher amplitudes of vibration.

In terms of methodology of analysis of response to crowd loads, possibility of occurrence of feedback phenomena must be evaluated first. The simple formula (1) was derived in the context of the London Millennium footbridge studies [3] and can be used to provide an estimate of the required number of pedestrians N_L to trigger synchronisation.

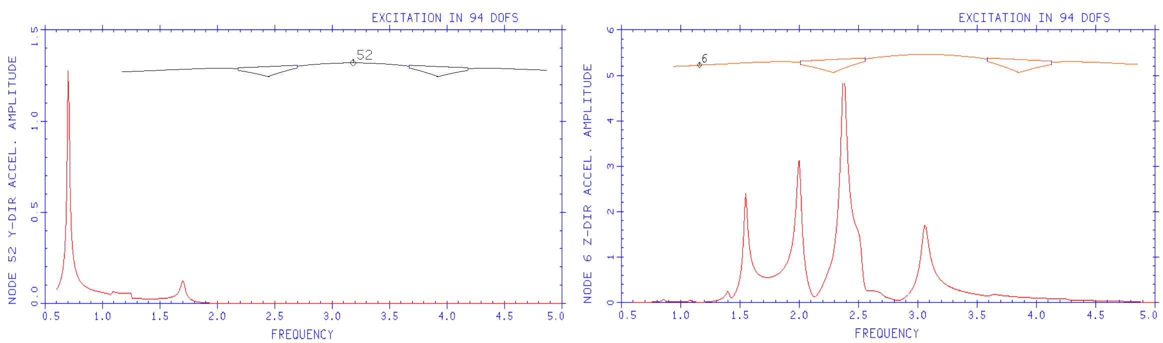
$$N_L = \frac{8\pi\xi M}{k} \tag{1}$$

Equation (1) assumes that pedestrians are uniformly distributed along the footbridge of modal mass M and damping ratio ξ and that the investigated vibration mode is sinusoidal and normalised to unit. Constant k is defined as 300Ns/m for a vibration frequency in the range 0.5 to 1 Hz. This equation can be used also to estimate the required damping of the footbridge to ensure that, for a specified density of pedestrians, synchronisation does not occur. If synchronisation is not expected, a random vibration response can be assumed. If synchronisation is expected, the bridge response must be evaluated by specifying a certain level of pedestrian synchronisation.

For the calculation of the response due to a flow of pedestrians, simplified formulae can be employed to obtain enhancement factors associated with a single pedestrian, according to the expected degree of correlation. Alternatively, if the action of one pedestrian is characterised by the dominant frequency component, a spectral analysis can be performed, in which case the bridge is loaded with the defined density of pedestrians considering a specified correlation or synchronisation factor. A more sophisticated approach is based on the simulation of pedestrian flow by concentrated moving loads generated according to a specified law. The stochastic nature of the spatial distribution of pedestrians, of the corresponding pacing rates and of the associated phase implies the need to develop multiple simulations to characterise the response for each critical vibration mode.

For the present footbridge, formula (1) states that 145 pedestrians are required to induce synchronisation in correspondence with the first lateral vibration mode. Considering an approximately sinusoidal mode defined in an equivalent length of 174m, that corresponds to 0.2 person/m^2 uniformly distributed along that length centred at the mid-span of the bridge, under which the bridge would be unstable for the first lateral mode if no control measures are taken. Furthermore, to guarantee no synchronisation exists for a uniform distribution of 1 person/m^2, a 5% damping ratio would be required.

The spectral approach was then taken to evaluate the response to a continuous flow of pedestrians, based on a distribution of 1 person/m^2 applied in criteriously defined lengths of the bridge and on 20% synchronisation. This same approach was used for the analysis of the response associated with vertical vibration modes. Figure 6 shows plots of the spectral acceleration along lateral and vertical direction, at mid-span and at the middle of the transition span, respectively, considering the corresponding lateral and vertical loading along the entire bridge. Table 4 resumes the maximum response expressed in terms of displacements and accelerations, d_{fmax} and a_{fmax}, respectively. The analysis of this table shows very high levels of displacement and acceleration for various vibration modes.



(a) *(b)*

Fig. 6 - Spectral acceleration associated uniformly distributed pedestrian load (1person/m2): (a) lateral acceleration at mid-span; (b) vertical acceleration at mid-point of transition span (m/s^2)

4.3 Definition of vibration limits

Load models associated with pedestrian actions are disputed, model uncertainties are inevitable, but one important difficulty refers to the specification of vibration limits.

Existing codes and literature provide limit accelerations associated with the response to a single pedestrian, only. BS5400 sets $0.5\sqrt{f}$ as maximum vertical acceleration, where f is the bridge natural frequency, and Eurocode 5 sets 0.2m/s^2 as maximum horizontal lateral acceleration for timber footbridges under normal use. Some authors, like Kreuzinger [4], suggest that the same limits are applied to groups of persons or to crowd loading. In terms of feedback phenomena, the trigger value of 10mm defined by Bachmann [3] can be adopted.

These limits were calculated for the current footbridge and are shown in the last column of Table 4. Yet, conclusions and selection of modes to be controlled require careful insight into those values. Although specified limits to values that are intolerable or just disturbing to pedestrians are very close to each other and some degree of subjectivity exists in the definition of those limits, vertical vibrations beyond 1.8m/s^2 or lateral vibrations higher than 0.7m/s^2 are clearly not acceptable to pedestrians. And though there is a higher variation in the acceleration limits, vertical accelerations of the order of 1m/s^2 or lateral accelerations of about 0.3m/s^2 are definitely alarming. Anyway, it is quite unreasonable to define the limit accelerations independently of the bridge utilisation intensity.

Table 4 - Bridge maximum response associated with the excitation by a group of pedestrians or a crowd. Acceleration limits

Mode number	Natural frequency (Hz)	Antinode	d_{gmax} (N=13) (mm)	a_{gmax} (N=13) (cm/s^2)	d_{fmax} (1p/m^2) (mm)	a_{fmax} (1p/m^2) (cm/s^2)	a_{max} (BS5400)/ (EC5)(cm/s^2)
1	0.704	52Y	8	21	66	128	42 / 20
4	1.403	32Y	3	24	8	62	59 / 20
5	1.548	17Z	10	94	41	389	62
7	2.001	6Z	7	110	20	314	71
8	2.261	56Z	2	33	3	57	75
10	2.375	6Z	10	205	23	511	77
11	2.498	33Z	8	193	21	490	79
12	2.522	71Z	11	255	31	767	79
13	2.705	51Z	1	43	4	119	82
14	3.060	16Z	2	65	7	267	87

4.4 Summary of results

Based on the above defined vibration limits and considerations and from results summarised in Tables 3 and 4, the following conclusions are drawn:

- Acceleration limits associated with the action of a single pedestrian are satisfied by all vibration modes;
- The synchronised action of a group of 13 pedestrians generates acceptable lateral accelerations but vertical accelerations exceed the BS5400 limits in several of the vertical vibration modes (5, 7, 10, 11 and 12). However, since full synchronisation of 13 pedestrians is very unlikely, except in a situation of intentional excitation, in which case comfort limits should no longer be imposed, only modes 10, 11 and 12 are considered relevant for the purpose of control measures;
- The crowd loading response exceeds the limit values in all modes, except in mode 8. For the lateral direction, mode 1 is very critical because of the very high levels of displacement and acceleration in its frequency. The other lateral mode (mode 4) is not considered critical because 20% synchronisation at a pacing rate of 2.8Hz seems unlikely. The remaining vertical modes (5, 7, 10, 11, 12, 13 and 14) are associated with very high accelerations. Some of these modes (10, 11 and 12) are also responsible for significant displacements, meaning that synchronisation can occur.

Taking into account the various uncertainties discussed above and the central location and expected intensity of use of the bridge, it is anticipated the introduction of control measures for the eight modes 1, 5, 7, and 10 to 14. A final decision on the installation of the corresponding eight tuned mass dampers (TMDs) will be taken after construction and dynamic testing of the complete footbridge.

5. Design of control devices

5.1 Characteristics

The significant number of vibration modes to control would suggest the installation of viscous dampers. However, the compact shape of the cross-section and continuity of the bridge put off such an option. Therefore, a control strategy based on the installation of eight TMDs was favoured.

TMDs were designed from natural frequencies of the bridge loaded with the frequent value of crowd loads, following the procedure described by Bachmann and Weber [6]. The structure was represented by a single degree of freedom (SDOF) system with modal properties equivalent to those of the controlled vibration mode, and optimal characteristics of mass M_T, stiffness K_T and damping C_T of a TMD to install at the closest location to the antinode of the vibration mode to control were defined. Table 5 shows the TMDs characteristics following the choice of a ratio μ between the mass M_T and the modal mass M of 2% for all modes. Total mass of the TMDs is 33000kg, representing 3.8% of the unloaded bridge mass.

5.2 Analysis of efficiency

The expected efficiency μ_a of the proposed TMDs, defined by the ratio between the uncontrolled and the controlled maximum accelerations, was calculated on the basis of the modelling of each mode as a SDOF system and is of the order of 5, assuming perfect tuning, which means that the calculated accelerations are expected to reduce about 5 times. The maximum displacements and accelerations of the controlled system are represented in Table 5.

It was questioned whether the efficiency of the combined TMDs installed in the footbridge is similar to the efficiency obtained by separate simulation of each TMD at a time. Therefore, a new model was constructed, in which concentrated masses with associated springs and viscous dampers were included, with characteristics identical to those specified in Table 5 for the TMDs. A new set of natural frequencies and modal shapes were obtained. By defining equivalent damping ratios to each of the new modes, the spectral response of the new controlled system to previously defined crowd loads was calculated and is summarised in the last two columns of Table 5. Some differences to the preliminary calculation are evident, especially for the mode shapes with very close natural frequencies.

In terms of the final response of the controlled footbridge, it is noted that for some of the modes (5, 10, 11 and 12) the BS5400 limits are exceeded considering crowd loading and 20% synchronisation. However, it is clear that the amplitudes of vertical acceleration are in the order of 1m/s^2, which is acceptable under the applied exceptional excitation. Moreover, calculations assume that 20% synchronisation occurs, although the reduction in displacements lessens the likelihood of such occurrence.

Table 5 - Characteristics of TMDs and estimate of response of the controlled footbridge

Mode	Frequency (Hz)	M_{TMD} (kg)	K_{TMD} (N/m)	c_{TMD} (Ns/m)	Node	$a_{fmáx}$ TMD (1p./m^2)	$d_{fmáx}$ TMD (1p./m^2)	$a_{fmáx}$ TMD (1p./m^2) *	$d_{fmáx}$ TMD (1p./m^2) *
1	0.704	4920	92399	3582	52Y	27	12	23	12
5	1.548	3860	350985	6184	17Z	71	7	73	9
7	2.001	3380	513535	6999	6Z	57	3	82	7
10	2.375	1582	338605	3888	5Z	95	4	107	3
11	2.498	2260	535122	5842	33Z	89	3	128	4
12	2.522	2040	492357	5324	71Z	140	5	95	5
13	2.705	9700	2693182	27154	51Z	22	1	30	5
14	3.060	5560	1975500	17607	16Z	48	1	32	1

*Spectral evaluation based on numerical simulation of the ensemble of TMDs installed in the footbridge

6. Final notes

A dynamic study for assessment of pedestrian induced vibrations on the Coimbra footbridge and for the design of control devices has been described, emphasizing the various levels of uncertainty that are present. The efficiency of a proposed control solution has been investigated. The bridge is currently under construction and will be extensively tested. Initially, modal properties of the bridge will be adjusted and the design of the TMDs will be calibrated. Then, TMDs will be tuned. Finally, the footbridge will be monitored during its first year in service in order to assess potential episodes of vibration.

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